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Investigating Solutions to Non-Routine Mathematics Problems in a Collaborative Setting

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Abstract

The study investigated Grade 9 students' solutions to non-routine mathematics problems (NRPs) in a collaborative learning setting. The implementation aimed to enhance learners' problem-solving proficiency and collaborative engagement through structured NRPs aligned with the Most Essential Learning Competencies (MELCs) prescribed by the Department of Education (DepEd), Philippines for the first quarter, particularly on solving quadratic equations using methods such as completing the square and applying the quadratic formula. Specifically, the study sought to (1) develop non-routine problems and (2) examine students' strategies in solving algebraic problems. A mixed-methods design was employed involving twenty-five (25) Grade 9 learners from TNHS, a public school in Lanao del Sur. Findings consistently aligned with the study's objectives. First, in developing NRPs suited to Grade 9 learners, teachers' evaluation confirmed their validity and instructional relevance, with all five problems rated Excellent (overall mean = 3.47). These ratings highlighted strong content validity, clarity of presentation, and appropriateness to learners' cognitive readiness. Second, in examining learners' strategies and proficiency, group outputs showed steady progress from Beginning in NRP 1 ($\bar{x} = 2.45$) to Proficient in NRP 5 ($\bar{x} = 3.80$), with an overall mean of 3.42 (Approaching Proficiency). Classroom observations reinforced this trajectory, rising from Apprentice in NRP 1 ($\bar{x} = 3.27$) to Proficient in NRP 5 ($\bar{x} = 4.87$), though some groups continued to struggle with generalizing solutions and systematically recording procedures. Qualitative insights further confirmed that learners demonstrated flexible reasoning, increased confidence in non-routine problem solving, and stronger engagement during collaborative work. Overall, the integration of contextualized NRPs aligned with MELCs on quadratic equations effectively enhanced students' mathematical proficiency, problem-solving confidence, and collaborative participation, thereby achieving both research objectives and demonstrating the pedagogical value of non-routine problems in fostering higher-order thinking among Grade 9 learners.

Keywords: non-routine problems; collaborative learning; quadratic equations; problem-solving strategies; mathematics education

1. Introduction

Mathematics has long been recognized as a foundational discipline that cultivates learners' critical thinking, reasoning, and problem-solving abilities. However, most mathematics classrooms in the Philippines and around the world have traditionally emphasized routine problem solving, wherein students depend on memorized formulas and procedural applications. This overemphasis on procedural fluency often leads to superficial learning, limited creativity, and weak collaboration among students (Nguyen et al., 2020).

In response, recent educational reforms and research have underscored the need to integrate non-routine problem solving (NRP) into mathematics instruction. Unlike routine exercises with fixed solutions, NRPs present unfamiliar, open-ended situations that require learners to think flexibly, explore various strategies, and reason logically through collaboration. Studies consistently show that engaging students in NRPs promotes deeper conceptual understanding, enhances metacognitive awareness, and develops higher-order thinking skills necessary for real-life mathematical applications (Keleş & Yazgan, 2025). Moreover, learners' engagement with NRPs fosters persistence, reflective thinking, and confidence in mathematical reasoning, thereby supporting both cognitive and affective dimensions of learning (Boaler, 2016).

The shift toward non-routine mathematical tasks also highlights the importance of collaboration as a key 21st-century learning skill. Collaborative learning environments allow students to communicate ideas, assume flexible roles, support team dynamics, and resolve conflicts—competencies that are essential for success in both academic and professional settings (Felmer, 2023). Through interaction and shared inquiry, students construct meaning collectively and refine understanding through dialogue. For junior high school learners, particularly those in Grade 9 where algebraic concepts become more abstract, collaboration serves as a valuable scaffold for comprehension and confidence building. Working in groups enables learners to exchange strategies, clarify misconceptions, and co-construct mathematical understanding, thereby strengthening both cognitive and social competencies (Junaid, 2025).

In the context of Tugaya National High School (TNHS), classroom observations revealed that many students struggle with solving non-routine algebra problems, especially when working individually. Learners often rely on rote memorization of formulas and procedures, showing limited capacity to transfer or adapt their knowledge to unfamiliar situations. Furthermore, collaborative classroom interactions at TNHS often displayed uneven participation—some students dominated discussions while others remained passive or disengaged. These findings align with Johnson and Johnson (2019), who noted that without structured collaboration, group work may fail to yield cognitive and social benefits. Such evidence underscores the need for structured approaches that integrate non-routine problem solving with collaborative learning to enhance both engagement and problem-solving proficiency in mathematics.

Taken together, the literature establishes that non-routine mathematics problem solving is integral to developing higher-order thinking skills, with success depending on students' strategic flexibility, confidence, and collaborative interactions. These insights frame the present study's investigation into how Grade 9 students collaboratively approach non-routine algebra problems, aiming to extend understanding of effective strategies and group dynamics in mathematical problem solving.

To address these challenges, the present study implemented and evaluated non-routine problem (NRP) activities in a collaborative classroom setting. Using the classroom observations and performance evaluations, the study examined the strategies used by Grade 9 students in solving non-routine algebra problems. The findings of this research aim to provide empirical evidence that collaboration and problem solving can reinforce one another, demonstrating that NRPs are not only effective tools for improving mathematical understanding but also for cultivating essential teamwork and communication skills among junior high school learners. The primary aim of this study was to investigate Grade 9 students' solutions to non-routine mathematics problems in a collaborative learning environment. Specifically, it sought to develop non-routine problems in mathematics suited to Grade 9 learners and to examine the strategies students used while solving non-routine algebra problems in a collaborative setting.

2.Methodology

2.1 Research Design

This study adopted a descriptive research design employing both quantitative and qualitative approaches. Its main objective was to develop and implement non-routine mathematics problems designed to enhance Grade 9 students' problem-solving and collaborative skills. Descriptive statistics such as frequency, percentage, and weighted mean were used to analyze teachers' evaluations and learners' performance, while qualitative data from interviews was thematically analyzed to capture students' strategies and collaborative experiences.

2.2 Research Setting and Participants

The study was conducted at Tugaya National High School (TNHS), a public secondary school in Tugaya, Lanao del Sur, Philippines. The school provides a diverse learning environment suitable for implementing non-routine problem (NRP) activities in mathematics. The participants were Grade 9 students, selected for their readiness to engage in complex problem-solving tasks and collaborative mathematical thinking. In addition, three mathematics teachers participated to validate the developed NRPs and observe their classroom implementation. All phases of the study adhered to ethical research standards, ensuring informed consent, voluntary participation, and confidentiality.

2.3 Research Instrument

The study employed multiple instruments to gather data:

1) Teacher Evaluation Rating Sheet. Three mathematics teachers validated the developed non-routine problems (NRPs) using criteria that assessed higher-order thinking, reasoning, clarity, multiple solution strategies, and collaborative potential.

2) Students' Non-Routine Problem Outputs. Five NRPs on quadratic equations were implemented, and students' written solutions were assessed using a rubric based on the National Council of Teachers of Mathematics (1989) indicators for problem-solving: Understanding the Problem, Planning a Solution, Carrying Out the Plan, and Looking Back.

3) Observation Checklist. Classroom interactions and group dynamics during NRPs were recorded using a structured checklist covering focus, work quality, strategy appropriateness, problem exploration, solution extension, and procedure recording.

4) Pre- and post-implementation interviews and FGD explored students' experiences, strategies, and perceptions of collaboration during NRPs.

2.4 Data Gathering Procedure

The first part of the study adhered to MSU-IIT's ethical guidelines. Permission to conduct the study at Tugaya National High School, a public secondary school located in Tugaya, Lanao del Sur, serving junior and senior high school learners from the surrounding communities, was secured from the school principal. Informed consent forms were obtained from 25 Grade 9 students and their parents/guardians. The Non-Routine Problems (NRPs), which focused on quadratic equations, were developed and validated by three mathematics teachers to ensure clarity, appropriateness, and alignment with curriculum standards. The NRPs were implemented over five days, with students organized into five groups of five members. One problem was given per day, and students worked collaboratively under the researcher's facilitation. During implementation, data was collected through group outputs on NRPs, classroom observations, and interviews to capture learners' problem-solving strategies and collaboration skills. Finally, all collected data were analyzed to evaluate the effectiveness of the NRP implementation in enhancing students' collaborative problem-solving abilities.

2.5 Data Analysis

This study employed a mixed-methods approach, combining both quantitative and qualitative analyses to investigate Grade 9 students' solutions to non-routine mathematics problems (NRPs) in collaborative settings and to examine the impact of NRPs on their collaborative skills. The quantitative data were analyzed using descriptive statistics, guided by the study's objectives. Frequencies and percentages were used to describe the observation checklists, providing a clear picture of how often specific collaborative behaviors occurred and the proportion of learners who demonstrated them. Data from the Non-Routine Problem (NRP) instruments were summarized using means and percentages, and these were interpreted following the Department of Education (DepEd) proficiency levels as outlined in DepEd Order No. 73, s. 2012.

The evaluation of the developed NRPs was guided by a rubric, with mean ratings interpreted using intervals. Table 1 presents the results of this rubric, showing how the NRPs were rated from "Needs Improvement" to "Excellent" depending on the computed mean scores.

Table 1: Evaluation of the Developed NRP

Intervals	Interpretation
3.26-4.0	Excellent
2.51-3.25	Very Good
1.76-2.50	Good
1.0-1.75	Needs Improvement

The performance of groups in solving NRPs was also evaluated using proficiency levels based on DepEd norms. Table 2 shows the interpretation of group scores, ranging from "Beginning" to "Advanced." The scale converts raw scores into equivalent numerical values and proficiency levels, with the highest possible score being 16 points and the lowest being zero.

Table 2: Evaluation of the Group' Scores on NRP

Intervals	Equivalent Numerical Value	Level of Proficiency
4.00	90% and above	Advanced
3.50–3.99	85%-89%	Proficient
3.00–3.49	80%-84%	Approaching Proficiency
2.50–2.99	75%-79%	Developing
1.00–2.49	74% and below	Beginning

In addition, the observation checklist was evaluated to measure learners' collaborative skills. Table 3 presents the interpretation scale, which ranges from "Novice" to "Proficient." This scale captures learners' progression from minimal mastery of collaborative skills to a well-developed ability to perform activities effectively in group settings.

Table 3: Evaluation of Observation Checklist

Intervals	Interpretation
1.00 to 1.79	Novice
1.80 to 2.59	Novice / Apprentice
2.60 to 3.39	Apprentice
3.40 to 4.19	Apprentice / Proficient
4.20 to 5.00	Proficient

To complement the numerical findings, qualitative data were analyzed using thematic analysis. Interviews were transcribed verbatim, coded, and examined to identify recurring patterns in students' problem-solving strategies, collaboration experiences, and reflections on teamwork. Thematic analysis of interview data identified three central themes: NRP Application, which highlighted how learners approached and applied strategies in solving non-routine problems; Empowering Learning, which emphasized how NRPs fostered confidence, active participation, and peer support; and Learning Experiences, which captured students' reflections on teamwork, communication, and challenges such as conflict resolution.

By integrating descriptive statistics with thematic analysis, the study provided a holistic understanding of learners' collaborative skills and problem-solving performance. Quantitative data offered objective measures of proficiency, while qualitative findings explained the context and reasoning behind observed behaviors. Together, these analyses allowed for triangulation, strengthening the validity of results and offering a nuanced perspective on how NRPs foster collaboration and problem-solving among Grade 9 learners.

3. Results and Discussion

The results are organized into three primary sections: the development of Non-Routine Problems (NRP), the pre-evaluation of these tasks, and their implementation within a collaborative setting.

3.1 Development of Non-Routine Problems (NRP)

A total of five Non-Routine Problems (NRP) were developed on the topic Solving Quadratic Equations by Completing the Square and the Quadratic Formula. Of these, two (NRP Nos. 2 and 3) applied the method of completing the square, while three (NRP Nos. 1, 4, and 5) required the quadratic formula. The development followed specific criteria emphasizing higher-order thinking, reasoning, creativity, and collaboration. Each problem underwent revisions from the first to the final draft based on expert feedback and teacher evaluation. The final versions were designed to promote multiple solution strategies, demand reasoning beyond simple computation, and encourage creativity and group discussion. Moreover, the problems were intentionally challenging, required sufficient time to solve, and connected mathematical concepts to realistic contexts, ensuring alignment with the objectives of developing learners' critical, innovative, and reflective thinking skills.

The first Non-Routine Problem (NRP) was developed to introduce a geometric modeling situation that integrates algebraic reasoning and spatial visualization. Table 4 presents the developmental process from the initial to the final draft, showing how revisions were made to enhance the problem's level of reasoning, contextual relevance, and alignment with higher-order thinking objectives.

Table 4: Developing Non-Routine Problem No. 1

First draft	**Final draft**
Problem: A rectangular piece of cardboard has a length that is 8 cm longer than its width. Squares measuring 4 cm are cut from each corner and the sides are folded up to form an open box. Determine the dimensions of the box.	Problem: An open box is to be formed from a rectangular piece of cardboard whose length is 8 cm longer than its width. To form the box, a 4 cm square is removed from each corner. The edges are then turned up to form the box. The box must hold at least 448 cm^3. Find the dimensions of the box.

The first draft of Non-Routine Problem No. 1 focused on forming an open box from a rectangular sheet of cardboard. While it required students to apply algebraic relationships between length, width, and area, it remained largely procedural in nature. The absence of a constraint limited the opportunity for exploration and reasoning. To strengthen the problem's cognitive demand, the final draft introduced a volume requirement of at least 448 cm^3, transforming it into a non-routine modeling problem. This revision encouraged students to connect geometric understanding with algebraic reasoning and to explore multiple possible solutions. It also increased real-world relevance, as students needed to determine feasible dimensions for an actual container.

This improvement followed specific developmental criteria, namely: Higher-order thinking, by requiring analysis and justification of feasible results; Authentic context, by grounding the problem in a realistic box construction scenario; Multiple strategies, as students could use algebraic or graphical reasoning; and Collaborative problem-solving, promoting peer discussion on viable dimensions. According to the teachers' evaluation, Non-Routine Problem No. 1 obtained an overall mean of 3.37 (Excellent). It was rated highest in the criteria "Increase reasoning ability" (M = 3.67) and "Encourage more than one solution and strategy" (M = 3.67), validating that the revisions enhanced analytical reasoning and strategic flexibility. The criterion "Require higher-order thinking skills" (M = 3.00), rated Very Good, shows that while the problem posed a challenge, it remained appropriate for Grade 9 learners.

Table 5: Developing Non-Routine Problem No. 2

First draft	Second draft	Final draft
Problem: A company recorded the average weekly income of its employees from 2006 to 2012. The data can be modeled by the equation $I = 0.18n^2 + 6.48n + 3240$ where I is the income in pesos and n is the number of years since 2006. Find the income for the year 2012.	Problem: A company modeled its employees' average weekly income (in pesos) over several years using the formula: $I = 0.18n^2 + 6.48n + 3240$, where n represents the number of years since 2006. If the income in one year was ₱3,268.80, find the value of n. Use an algebraic method to determine which year this occurred.	Problem: A company tracked its employees' average weekly income from 2006 to 2012. They discovered the income (in pesos) could be estimated using the formula: Income = $0.18n^2 + 6.48n + 3240$, where 'n' represents the number of years since 2006. In one particular year, the average weekly income was exactly ₱3,268.80. Using the method of completing the square, determine which year this occurred. Show all your steps.

The second Non-Routine Problem (NRP) was constructed to link quadratic modeling to a real-world context (employee income over time) and to provide learners with experience in inverting a model (finding the independent variable given an output). Table 5 displays the evolution of the item from the first draft through the final draft.

The development of NRP No. 2 progressed from a routine substitution task to a more cognitively demanding inversion problem. The first draft required only the forward evaluation of a quadratic model, while the second reframed the task as determining the year corresponding to a given income, thereby requiring learners to solve a quadratic equation. The final draft further aligned with instructional goals by explicitly requiring the use of completing the square, ensuring targeted practice of this method while situating the task in an authentic income-based context. These revisions reflect key development criteria, including promoting higher-order thinking, strengthening procedural proficiency, encouraging realistic interpretation, and allowing multiple solution pathways despite the specified method.

Teachers rated NRP No. 2 with an overall mean of 3.26 (Excellent). Item-level results further clarified its strengths: “Produce creative and innovative students” (4.00), “Use answers and procedures that are not immediately clear” (3.67), and “Encourage more than one solution and strategy” (3.67), indicating that the inversion task was engaging and afforded multiple approaches. Meanwhile, “Require higher-order thinking skills” (3.00) and “Challenging thinking skills” (3.00) reflected that the task, while improved, remained moderately accessible. The explicit instruction to use completing the square ensured focused practice but may have reduced perceived openness, consistent with the pattern of scores. Overall, NRP No. 2 successfully transitioned from a routine evaluation exercise to a meaningful inversion problem that strengthened procedural competence and model interpretation, with teacher feedback suggesting that an extension comparing alternative solution methods could further increase its cognitive demand.

The third Non-Routine Problem (NRP) was designed to deepen students’ conceptual understanding of quadratic equations by exploring the relationship between coefficients and roots. Table 6 presents the developmental process from the first to the final draft, illustrating how the task evolved from a procedural computation exercise into a reasoning- and explanation-based exploration.

The development of Non-Routine Problem No. 3 reflects a deliberate progression from procedural computation toward deeper analytical reasoning. The first draft required students to solve two quadratic equations and compare numerical results, a task that emphasized symbolic manipulation but offered limited conceptual challenge. The second draft improved this by shifting the focus toward conceptual prediction—inviting students to reason about the equivalence of roots without performing explicit computations. The final draft advanced this further by incorporating layered reasoning tasks, asking learners to predict outcomes when all coefficients were negated and to analyze cases where only one coefficient changed sign. These refinements required students to generalize relationships among algebraic symbols, explore the effects of coefficient transformations, and connect symbolic and graphical interpretations, thereby aligning with the criteria of promoting higher-order thinking, encouraging multiple strategies, maintaining mathematical authenticity, and supporting collaborative discussion.

Table 6: Developing Non-Routine Problem No. 3

First draft	Second draft	Final draft
Problem: Given the quadratic equation $-2x^2 + 8x + 3 = 0$, find its roots. Then, change the signs of all the coefficients to create a new equation $2x^2 - 8x - 3 = 0$. Solve this new equation and compare the two sets of solutions. Are they the same or different? Explain why.	Problem: The quadratic equation $-2x^2 + 8x + 3 = 0$ can be transformed by changing the sign of all its coefficients, resulting in $2x^2 - 8x - 3 = 0$. Without solving, predict whether the two equations will have the same set of roots or not. Justify your reasoning based on how changing the signs of coefficients affects the equation and its graphical representation.	Problem: Consider a quadratic equation in standard form, $ax^2 + bx + c = 0$. Equation 1: The coefficients a, b, and c are $-2, 8$, and 3, respectively. Now, imagine you accidentally change the sign of all the coefficients, creating a new equation: $2x^2 - 8x - 3 = 0$. Without fully solving either equation, can you determine if the solutions to the two equations will be the same? Explain your reasoning. Think about how the relationship between the coefficients affects the solutions. If you only changed the sign of $'\boldsymbol{b}'$, would the solutions be the same? Explain. If you only changed the sign of $'\boldsymbol{c}'$, would the solutions, be the same? Explain.

Teachers' evaluation affirmed the strength of the final version, with NRP No. 3 receiving an overall mean of 3.52 (Excellent). The highest ratings were for "Encourage more than one solution and strategy" (4.00) and "Produce creative and innovative students" (4.00), confirming that the open-ended design effectively fostered flexibility and creativity. The criterion "Require higher-order thinking skills" (3.67, Excellent) further reflected recognition of the cognitive challenge embedded in analyzing coefficient–root relationships. Meanwhile, the ratings "Use answers and procedures that are not immediately clear" (3.00) and "Challenging thinking skills" (3.00) indicated that the task remained appropriately accessible for Grade 9 learners. Overall, the evolution of NRP No. 3 demonstrates a purposeful shift from computation to conceptual reasoning, with teacher ratings validating its effectiveness in cultivating analytical thinking and deeper understanding of quadratic relationships.

Table 7 presents the development of Non-Routine Problem (NRP) No. 4, which centers on applying quadratic reasoning to a real-world measurement scenario. The problem situates learners in a design context where they must determine the dimensions of a car park given its area and the relationship between its length and width. This task evolved from a structured, procedural format in the first draft to a more exploratory and student-centered version in the final draft, encouraging deeper reasoning and engagement.

Table 7: Developing Non-Routine Problem No. 4

First draft	Final draft
Problem: A rectangular car park is being designed for a new building. The length of the car park is 12 meters longer than its width, and its total area must be 35 square meters. Determine the dimensions of the car park. Activities: 1. Represent the width of the car park as w. 2. Express the length in terms of w. 3. Write the equation representing the area of the car park. 4. Solve the equation to find the possible values of w and identify the realistic solution. 5. State the final dimensions of the car park.	Problem: A new car park is being built in town. We know a few things about it: • It's shaped like a rectangle. • It's quite long! It's 12 *meters* longer than it is wide. • The total area of the car park needs to be 35 *square meters* so enough cars can park there. Your Activities: a. If we call the width of the car park "*w*", how would you write down what the length is? (Remember, it's 12 meters longer than the width). b. Can you write an equation that shows how to calculate the area of the car park using "*w*"? (*Area = length x width*) c. Can you guess what the width and length of the car park might be? Try some numbers to see if you can get close to 35 square meters. What are the length and width of the car park?

The first draft of Non-Routine Problem No. 4 resembled a traditional procedural textbook exercise, directing learners to represent variables, form equations, and solve for unknowns through algebraic manipulation. Although mathematically correct, it was highly structured and left little room for exploration or learner autonomy. In contrast, the final draft adopted an inquiry-oriented design using more accessible language ("A new car park is being built in town") to increase engagement. Instead of immediately guiding students toward algebraic formulation, it allowed them to estimate, test, and revise guesses before formalizing equations, transforming the task into an exploratory modeling activity that encouraged intuitive reasoning and multiple solution paths.

This transformation aligned with the criteria for developing non-routine problems, promoting higher-order thinking, authentic context, flexible strategies, and collaborative problem-solving. Teachers' evaluation supported this improvement, with NRP No. 4 receiving an overall mean of 3.45 (Excellent). The criteria "Increase reasoning ability" (M = 3.67) and "Encourage more than one solution and strategy" (M = 3.67) received the highest ratings, indicating strong effectiveness in stimulating critical and flexible thinking. The criterion "Require higher-order thinking skills" earned a mean of 3.33 (Very Good), showing that while the task remained accessible, it successfully required learners to engage in reasoning beyond routine computation.

The fifth Non-Routine Problem (NRP) was designed to connect quadratic reasoning to a practical, vocational context (plywood cutting). Table 8 displays the progression from a brief, procedural first draft to a final draft that emphasizes interpretation, realism, and justification.

Table 8: Developing Non-Routine Problem No. 5

FIRST DRAFT	SECOND DRAFT	FINAL DRAFT
Problem: Mr. Bonifacio needs to cut rectangular plywood for his furniture. Plywood 1: The length is the width plus 2 ft. The area is 15 ft². Find the width and length. Plywood 2: The length is one more than the width. The area is 2 ft². Find the width and length. Activities: 1. Write an equation with w for the width for each plywood. 2. Solve for w. 3. Find the corresponding length. 4. Show your calculations.	*Problem*: Mr. Bonifacio needs two rectangular plywood pieces for his furniture. For each piece, determine width and length. Plywood 1: Length = width + 2 ft; area = 15 ft². Plywood 2: Length = width + 1 ft; area = 2 ft². Activities: 1. Write and solve the equation for w for each plywood. 2. For each solution set, determine whether all algebraic roots are physically meaningful; explain why negative roots (if any) are rejected. 3. Discuss whether the computed dimensions are practical for furniture construction (consider whole-number dimensions, typical plywood sizes, and potential waste from cutting). 4. Show your work and justify your answers.	*Problem*: Mr. Bonifacio needs to cut some rectangular plywood for his furniture. Let's help him figure out the sizes! Plywood 1: • The length is the width plus 2 ft. • The area is 15 $sq\ ft$. • Your job: Find the $width\ and\ length$. Plywood 2: • The length is one more than the width. • The area is 2 $sq\ ft$. • Your job: Find the $width\ and\ length$. Questions: 1. Write the Equation: For each plywood, write an equation using "w" for width that shows how to find the area. 2. Find the Width: Solve your equation to find the width of each plywood. 3. Find the Length: Use the width you found to figure out the length of each plywood. 4. Show Your Work: Explain how you found the length and width for each plywood.

The first draft of NRP No. 5 consisted of two direct quadratic setups that required only algebraic solution, emphasizing procedural fluency without engaging students in interpretation or real-world application. In the revised and final versions, explicit prompts were added to require learners to judge the physical meaningfulness of algebraic roots and evaluate practicality based on typical plywood sizes and potential waste. These revisions shifted the task from pure computation toward mathematical modeling, encouraging students to construct the quadratic model, solve it, interpret acceptable roots, and determine whether the solutions make sense in a woodworking context.

These changes align with the study's criteria for high-quality NRPs by strengthening higher-order thinking, promoting multiple strategies, providing an authentic carpentry scenario, and supporting collaborative reasoning. NRP No. 5 obtained the highest overall mean (M = 3.63, Excellent), with strong ratings in "Increase reasoning ability" (M = 4.00), "Use answers and procedures that are not immediately clear" (M = 4.00), and "Encourage more than one solution and strategy" (M = 4.00). Although "Require higher-order thinking skills" received a slightly lower rating (M = 3.00, Very Good), teachers still affirmed the task's effectiveness and accessibility for Grade 9 learners. Overall, the final draft effectively transformed a procedural exercise into a contextualized, interpretive task that deepens conceptual understanding and applied reasoning.

3.2 Pre-Evaluation of Developed NRP

The developed NRP was evaluated by Three (3) Mathematics educators who were also educators of the target school. The result of evaluation is presented in a table form. This section presents the results from the rubric used to evaluate the Non-Routine Problem (NRP) implementation. The mean ratings were interpreted using the intervals and descriptions. Ratings provided by three TNHS mathematics teachers; scale aligned with the Department of Education's official guidelines on assessment and rating of learning outcomes under the K to 12 Basic Education Curriculum (DepEd Order No. 73, s. 2012).

Table 9: Summary of Teachers' Evaluation of the Developed Non-Routine Problems (NRP)

Non-Routine Problem (NRP)	Overall Mean	Interpretation
NRP No. 1	3.37	Excellent
NRP No. 2	3.26	Excellent
NRP No. 3	3.52	Excellent
NRP No. 4	3.59	Excellent
NRP No. 5	3.63	Excellent
Overall Mean	3.47	Excellent

As presented in Table 9, the overall mean rating of the developed Non-Routine Problems (NRPs) was 3.47, which falls within the Excellent. This result indicates that the NRPs were consistently perceived by teacher-evaluators as high-quality instructional materials that effectively embody the essential characteristics of non-routine mathematical tasks. Among the five problems, NRP No. 5 obtained the highest mean score of 3.63, followed closely by NRP No. 4 with a mean of 3.59. These findings suggest that these tasks were particularly successful in engaging learners in complex reasoning, encouraging multiple solution strategies, and fostering creativity and collaboration. In contrast, NRP No. 2 received the lowest mean score of 3.26, though it still met the criteria for Excellent, reflecting the evaluators' consistent recognition of quality across all tasks.

The consistently excellent ratings across all NRPs affirm that the developed problems integrate critical elements of non-routine problem solving, including higher-order thinking, reasoning, creativity, and collaborative learning. These findings are supported by Nguyen et al. (2020), who emphasized that collaborative learning environments promote deeper understanding and retention of mathematical concepts through joint problem solving and knowledge construction. Similarly, highlighted that non-routine problems compel learners to move beyond memorized algorithms, thereby fostering analytical reasoning, creative thinking, and collaborative engagement.

Moreover, the high evaluation results imply that the developed NRPs are suitable for classroom implementation, particularly for fostering mathematical reasoning and problem-solving competencies among learners. This aligns with the observations of Junaid (2025), who noted that structured collaborative problem-solving activities enhance communication, teamwork, and creativity by exposing learners to diverse perspectives and encouraging interactive dialogue. The uniformity of high ratings across evaluators also suggests strong content validity and reliability, signifying that the tasks effectively meet pedagogical standards and intended learning outcomes.

In summary, the teachers' evaluations collectively demonstrate that the developed Non-Routine Problems (NRPs) are of excellent quality, effectively promoting cognitive challenge, creativity, and meaningful mathematical engagement among students. These results reinforce the broader literature on collaborative learning and non-routine problem solving, which consistently underscores their value in cultivating higher-order thinking, adaptive reasoning, and essential collaborative skills (Johnson & Johnson, 2019).

3.3 The Implementation of NRP in a Collaborative Setting

To examine specific strategies and performance of Grade 9 learners in solving non-routine algebra problems within a collaborative setting, five non-routine problem (NRP) activities were administered. These problems were carefully designed based on the developed evaluation criteria and aligned with the DepEd curriculum competencies. The NRP activities covered topics under Solving Quadratic Equations by Completing the Square (two problems) and Solving Quadratic Equations by the Quadratic Formula (three problems). Each activity required students to present a two-column solution showing their process and reasoning, allowing both problem-solving proficiency and collaborative engagement to be observed. The results from these activities provided insight into the learners' mathematical reasoning, teamwork dynamics, and application of various problem-solving strategies, forming the basis for assessing the impact of the implementation phase.

Table 10: Summary of Frequency and Percentage Distribution of Group Outputs on NRP 1-5
Participants: 25 Grade 9 students organized into five groups of five (N=5 groups)

Level of Proficiency Summary - NRP	Mean	Verbal Interpretation
NRP No. 1	2.45	Beginning
NRP No. 2	4.00	Advanced
NRP No. 3	2.85	Developing
NRP No. 4	4.00	Advanced
NRP No. 5	3.80	Proficient
Overall Mean	3.42	Approaching Proficiency

Legend (Proficiency Levels):
Beginning (74% and below): 1.00–2.49
Developing (75%–79%): 2.50–2.99
Approaching Proficiency (80%–84%): 3.00–3.49
Proficient (85%–89%): 3.50–3.99
Advanced (90% and above): 4.00

The results in Table 10 indicate that group performance varied across the five non-routine problems. The lower mean scores in Problem 1 (2.45, Beginning) and Problem 3 (2.85, Developing) suggest initial difficulty with unfamiliar and non-formulaic problem types. This finding is consistent with learners' pre-implementation reflections, where they expressed uncertainty when no direct formula was available. Such early struggles reflect reliance on procedural knowledge before developing deeper conceptual understanding, a challenge noted by Nguyen et al. (2020), who emphasized that non-routine problems require flexible reasoning and visualization strategies rather than repetitive procedural learning.

Performance improved markedly in later problems, with Problem 2 (4.00, Advanced) and Problem 4 (4.00, Advanced) demonstrating mastery, and Problem 5 (3.80, Proficient) showing strong proficiency. This progression illustrates growing adaptability and confidence as groups engaged more in collaborative discussions and strategy sharing. Such improvement resonates with Yazgan (2025), who highlighted the importance of strategic flexibility and metacognitive training in enhancing collaborative problem-solving potential across ability levels.

Post-implementation reflections further reinforce this interpretation. For instance, Group 3 noted that teamwork enhanced understanding and made complex problems more manageable: "Mas madali kapag nagtutulungan kasi mas maraming idea at mas mabilis mahanap ang sagot" (It is easier when we help each other because there are more ideas and the answer can be found faster).

Overall, the improvement across activities demonstrates that sustained engagement with non-routine, contextualized tasks promote not only mathematical reasoning but also cooperative learning. This conclusion is supported by Boaler (2016) who emphasized the importance of instructional design and teacher facilitation in scaffolding collaborative discourse and deepening conceptual understanding and affirmed that collaborative non-routine problem solving enhances communication, teamwork, empathy, and reflective thinking—skills indispensable for both academic success and broader social development.

Table 11: Summary of Observation of NRP Implementation- NRP 1-5

Classroom Observation on the Level of Proficiency	Weighted Mean	Verbal Interpretation
NRP No. 1	3.27	Apprentice
NRP No. 2	5.00	Proficient
NRP No. 3	3.57	Apprentice / Proficient
NRP No. 4	5.00	Proficient
NRP No. 5	4.87	Proficient

Legend of the Verbal Interpretation of the Weighted Mean:

1.00 to 1.79	Novice
1.80 to 2.59	Novice / Apprentice
2.60 to 3.39	Apprentice
3.40 to 4.19	Apprentice / Proficient
4.20 to 5.00	Proficient

Non- Routine Problem Prompts Referenced in Table 11:

NRP No. 1: An open box is to be formed from a rectangular piece of cardboard whose length is 8 cm longer than its width. To form the box, a 4 cm square is removed from each corner. The edges are then turned up to form the box. The box must hold at least 448 cm^3. Find the dimensions of the box.

NRP No. 2: A company tracked its employees' average weekly income from 2006 to 2012. They discovered the income (in pesos) could be estimated using the formula: Income $= 0.18n^2 + 6.48n + 3240$,

where 'n' represents the number of years since 2006.

In one particular year, the average weekly income was exactly ₱3,268.80. Using the method of completing the square, determine which year this occurred. Show all your steps.

NRP No. 3:

Consider a quadratic equation in standard form, $ax^2 + bx + c = 0$.

Equation 1: The coefficients a, b, and c are -2, 8, and 3, respectively.

Now, imagine you accidentally change the sign of all the coefficients, creating a new equation: $2x^2 - 8x - 3 = 0$.

1. Without fully solving either equation, can you determine if the solutions to the two equations will be the same? Explain your reasoning. Think about how the relationship between the coefficients affects the solutions.
2. If you only changed the sign of $'b'$, would the solutions be the same? Explain.
3. If you only changed the sign of $'c'$, would the solutions, be the same? Explain.

NRP No. 4: A new car park is being built in town. We know a few things about it:

- It's shaped like a rectangle.
- It's quite long! It's 12 meters longer than it is wide.

- The total area of the car park needs to be 35 square meters so enough cars can park there.

Your Activities:

- If we call the width of the car park "w", how would you write down what the length is? (Remember, it's 12 meters longer than the width).
- Can you write an equation that shows how to calculate the area of the car park using "w"? (Area = length x width)
- Can you guess what the width and length of the car park might be? Try some numbers to see if you can get close to 35 square meters.

What are the length and width of the car park?

NRP No. 5: Mr. Bonifacio needs to cut some rectangular plywood for his furniture. Let's help him figure out the sizes!

Plywood 1:

- The length is the width plus 2 ft.
- The area is 15 $sq\ ft$.
- Your job: Find the $width\ and\ length$.
- Plywood 2:
- The length is one more than the width.
- The area is 2 $sq\ ft$.
- Your job: Find the $width\ and\ length$.

Questions:

1. Write the Equation: For each plywood, write an equation using "w" for width that shows how to find the area.
2. Find the Width: Solve your equation to find the width of each plywood.
3. Find the Length: Use the width you found to figure out the length of each plywood.
4. Show Your Work: Explain how you found the length and width for each plywood.

As presented in Table 11, classroom observations revealed that learners initially performed at the Apprentice level in Problem 1 (3.27), indicating early challenges in adapting to non-routine tasks. This outcome reflects the tendency of students to rely on procedural approaches when confronted with unfamiliar problems, a difficulty noted in studies emphasizing the need for instructional designs that prioritize flexible reasoning over repetitive procedures (Nguyen et al., 2020).

Moreover, Performance improved considerably in later tasks, with Problem 2 (5.00, Proficient) and Problem 4 (5.00, Proficient) demonstrating mastery, and Problem 5 (4.87, Proficient) showing strong collaborative engagement. The progression from Apprentice to Proficient levels illustrates learners' growing adaptability and confidence in collaborative problem solving.

The observation data suggests that sustained exposure to non-routine problems enhances both mathematical reasoning and collaborative competence. The shift from Apprentice to Proficient levels across tasks highlights the effectiveness of structured non-routine activities in promoting higher-order thinking, communication, and teamwork—competencies essential for long-term learning and problem-solving success.

4. Conclusion and Recommendation

The study, Investigating Solutions to Non-Routine Mathematics Problems in a Collaborative Setting, examined Grade 9 students' approaches to solving non-routine mathematics problems (NRPs) within group contexts, focusing on problem-solving strategies and collaborative skills. Findings from classroom observations, proficiency summaries, interviews, and focus group discussions consistently demonstrated that the developed NRPs were effective in enhancing learners' reasoning, adaptability, and teamwork. While initial difficulties were observed in generalization and systematic recording, learners showed steady improvement, with later tasks reaching Proficient and Advanced levels of performance. Thematic analysis further revealed three central themes; NRP Application, Empowering Learning, and Learning Experiences; highlighting students' transition from uncertainty to strategic exploration, increased confidence through peer support, and greater enjoyment of mathematics through collaborative engagement. Overall, the implementation of NRPs in collaborative settings proved to be an effective pedagogical approach for fostering higher-order thinking, flexible problem-solving strategies, and meaningful social interaction among learners.

Based on these conclusions, it is recommended that teachers integrate non-routine problems regularly into mathematics instruction to help learners move beyond rote procedures and develop flexible strategies, while scaffolding metacognitive skills such as generalization and systematic recording through guided questioning, reflection journals, and structured group roles. School administrators and curriculum developers are encouraged to institutionalize NRPs within the MATATAG curriculum to strengthen higher-order thinking and collaboration, supported by sustained professional development in NRP design and facilitation. Additionally, resource banks of contextualized, real-life NRPs should be developed to ensure curriculum coherence and accessibility. Future researchers may extend this study to other subjects or grade levels, explore long-term retention of collaborative and problem-solving skills, and investigate the integration of digital collaborative tools to address time-management concerns and compare the effectiveness of collaborative versus individual approaches in fostering mathematical thinking.

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